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GENERAL REMARKS BEFORE THE MATHEMATICAL DESCRIPTION ACTIVE BASIC MODELS OF ECONOMIC DYNAMICS

ЗАГАЛЬНІ ЗАУВАЖЕННЯ ДО МАТЕМАТИЧНОГО ОПИСУ ДЕЯКИХ БАЗОВИХ МОДЕЛЕЙ ЕКОНОМІЧНОЇ ДИНАМІКИ

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Семенов А.С., Соколовська З.М., Будорацька Т.Л. Загальні зауваження до математичного опису деяких базових моделей економічної динаміки. Науково-методична стаття.
Стаття присвячена математичному підґрунтю досліджень динаміки економічних процесів на макрорівні. Стверджується необхідність формування нових підходів та пристосування існуючих методів і моделей до тенденцій сучасної економіки. Здійснено аналіз базових моделей економічної динаміки. Доводиться, що з математичної постановки класичних моделей економічної динаміки, наприклад, моделі Солоу, випливає експоненціальне зростання економіки. Результати, які випливають з постановки моделей, стосуються, зокрема, проблеми прогнозування криз. Запропоновано модифікацію математичної постановки деяких відомих моделей економічної динаміки з метою усунення неадекватного експоненційного зростання економіки та наближення результатів моделювання до динаміки реальних процесів.
Ключові слова: макроекономічні моделі, коректування математичних постановок, економічна динаміка, інтегральні залежності, диференціальні залежності

Semenov A.S., Sokolovska Z.M., Budoratska T.L. General Remarks Before the Mathematical Description Active Basic Models of Economic Dynamics. Scientific and methodical article.
The article is devoted to the mathematical basis of research on the dynamics of economic processes at the macro level. The necessity of forming new approaches and adapting existing methods and models to the trends of the modern economy is asserted. An analysis of the basic models of economic dynamics has been carried out. It is proved that from the mathematical formulation of classical models of economic dynamics, for example, the Solow model, exponential growth of the economy follows. The results that follow from the formulation of models concern, in particular, the problem of crisis forecasting. A modification of the mathematical formulation of some well-known models of economic dynamics has been proposed in order to eliminate the inadequate exponential growth of the economy and to bring the results of modeling closer to the dynamics of real processes.
Keywords: macroeconomic models, correction of mathematical statements, economic dynamics, integral dependencies, differential dependencies

The modern economy, as a complex socio-economic system, develops unstable: it is characterized by different modes of functioning – from intervals of stable development to chaotic behavior. Studying the dynamics of the behavior of any economic systems makes it possible to predict the prospects of their development, identify possible reserves, and apply measures to prevent negative phenomena.

The methodological apparatus of economic dynamics is the methods of mathematical analysis, differential and variational calculus, as well as mathematical modeling. There are a number of basic models designed to study the dynamics of economic processes at the macro level. Without diminishing the importance of well-known models of macrodynamics, such as the models of Solow, Phillips, Harrod-Domar, etc., it is necessary to emphasize the presence of flaws in their formulation, which definitely negatively affect the final results of the calculations. In the specialized literature, there are numerous discussions regarding the modernization of the mathematical formulations that are the basis of the above models.

However, the only point of view of specialists is the need to define new approaches, offer new methods, which will make macrodynamics models more flexible and adequate to modern economic realities.

This is especially relevant at the current stage of development of the world economy, which is in a state of significant transformations, increased entropy of the environment, changes in major trends and the appearance of significant challenges to further development.

Without pretending to provide an absolute solution to the problems, the authors only try to express their own comments on the mathematical description of some basic models of economic dynamics with the aim of further obtaining more adequate results based on them.

Analysis of recent researches and publications

The creation of classical models of macroeconomic dynamics was carried out by the efforts of many scientists, among whose works they have become classics [1-4]. In modern times, such domestic and foreign scientists as [5-10] made a significant contribution to the development of the theory and the creation of applied applications.

Let's consider some of the main theses inherent in these works.

The standard approach to the theory of continuous functions in natural science is to consider infinitesimally small increments of functions followed by a limiting transition. Such a limit transition makes it possible to summarize the analysis of the functioning of the analysis system of the solution of the differential equation. The formal transfer of this method to discretely changing functions-indicators of economic systems sometimes leads to both an incorrect mathematical model and a number of incorrect conclusions regarding the functioning of the economic system, for example, to the exponential growth over time of all indicators of the system (example - the Solow model) [6, 8].

Often there is an unjustified transition to dimensionless parameters, for example, to dimensionless time, and the construction of relationships between macroeconomic functions of different dimensions, while the equations of the model do not have "balanced" dimensions and self-modelity is absent (for example, the model of macroeconomic dynamics of Phillips [1]). The same lack of consistency of dimensions during optimization by the gradient method leads to "movement" to the optimum on the path of a non-collinear gradient.

In a simple formulation in the Solow model, the economy is considered as a single closed unstructured whole; produces one universal product that can be both consumed and invested.

There are five macroeconomic indicators:

indicators of the «flow» type, their values are accumulated during the year:

Y − gross domestic product (GDP);

I − gross investments;

C − consumption fund;

instantaneous variables:

K − main production funds;

L − the number of people employed in the production sphere;

The Solow model with discrete time is given by the system of equations:

$$Y_t = F(K_t, L_t), \tag{1}$$

$$Y_t = I_t + C_t, \tag{2}$$

$$K_t = (1-\mu) K_{t-1} + I_{t-1}, \tag{3}$$

$$L_t = (1+v) L_{t-1}, \qquad t=1,2,\dots T \tag{4}$$

where t=0 is the base year;

T − is the final year of the studied period;

K_0, L_0, I_0 – − given initial values.

The first equation defines GDP as a production function of resources; the second is the distribution of GDP into gross investments and consumption. The third is recurrent relations for determining the PPF, here μ is the coefficient of disposal (depreciation) of the PPF calculated per year. The fourth is the ratio for determining the number of employed. It is based on the hypothesis of the stability of the annual rate of increase in the number of employed v.

Here, equations (3) and (4) are linear dynamic elements. The model in Solow form is a dynamic structural diagram.

Solow model with continuous time.

Let us denote the rate of accumulation in the model by ρ, which is the share of the product used for investment, i. e.

$$I(t) = \rho Y(t), \quad C(t) = (1-\rho) Y(t),$$

and $0 < \rho < 1$ considering that investments are used to restore lost funds and increase them. For the increase of funds, the coefficient of disposal of funds μ calculated per year is introduced. Then the increase in funds ΔK is equal to:

$$\Delta K = K(t+\Delta t) - K(t) = \rho Y \Delta t - \mu K \Delta t.$$

Going to the limit, we have:

$$\frac{dK}{dt} = \rho Y - \mu K,$$

The increase in labor resources is proportional to the available labor resources L.

$$\Delta L = vL * \Delta t,$$

From here:

$$\frac{dL}{dt} = vL.$$

The solution of this equation:

$$L = L_0 e^{vt}$$

At $t \to \infty$ we see that $L \to \infty$. Naturally, this infinite labor resource cannot be realized. This discrepancy is a consequence of linearization.
So we have the following system of equations:

$$C(t) = (1-\rho)Y(t),$$

$$Y(t) = F(K, L),$$

$$L = L_o e^{vt},$$

$$\frac{dK}{dt} = \rho Y(t) - \mu K(t),$$

$$K_{(0)} = K_0.$$

You can enter the average labor productivity $y = {}^{Y}/_{L}$ and the average capital-labor ratio $k = {}^{K}/_{L}$.
Then, using the homogeneity property of the production function z, we have
$y = \frac{F(k,L)}{L} = F\left(\frac{K}{L}, 1\right) = F(k,1)$, which can be considered a function of one variable: $F(k, 1) = f(k)$.

We find the derivative

$$\frac{dk}{dt} = \frac{d}{dt}\left(\frac{K}{L}\right) = \frac{\rho Y - \mu K}{L} - \frac{K\nu}{L} = \rho y - (\mu + \nu)k.$$

So, let's come to the Cauchy problem for capital-labor ratio:

$$\begin{cases} \frac{dk}{dt} = \rho f(k) - (\mu + v)k \\ k(0) = k_0 = \frac{k_0}{l_0} \end{cases} \text{ nonlinear equation.}$$

If the capital-labor ratio is constant, then $\frac{dk}{dt} = 0$ the equation becomes algebraic $\rho f(k) - (\mu + v)k = 0$,
Here $f(k)$ − is an increasing function. The growth rate slows down at $k \to \infty$.
The economic indicators in such a situation are as follows:

dynamics of labor resources $L(t) = L_o e^{vt}$;

funds $K(t) = k_0 L(t) = k_0 L_o e^{vt}$;

the final product $Y(t) = f(k_0) L_0 e^{\nu t}$;

The amount of non-production consumption:

$$C(t)=(1-\rho)Y(t)=(1-\rho)\, f(k_0)L_o e^{vt};$$

Investment size $I(t)=\rho\, f(k_0)L_o e^{vt}$.

So: on a stationary trajectory, with constant funding, the main economic indicators grow indefinitely, exponentially, in proportion to labor resources (all in a limited period of time).

The purpose of the article is the analysis of the basic models of economic dynamics with the determination of approaches to their modernization in accordance with the modern realities of the economy, in particular, the determination of the rational distribution of the final product between the sphere of consumption and the expanded reproduction of capital

The main part

The Harrod-Domar model is described by analogy with the Solow model by ratios

$$Y(t)=C(t)+S(t);\quad S(t)=I(t);\quad S(t)=\mu Y(t);$$

$$\frac{d\,K(t)}{dt}=I(t);\qquad K(t)=\nu Y(t). \tag{5}$$

where Y(t) − is national income;

C(t), S(t) – annual volumes of consumption and accumulation;

I(t) – annual volume of investments;

K(t) – capital;

t − is dimensionless time measured in years;

$0 < \mu < 1$ and ν − are dimensionless constants (roughly, $\mu \sim 0.5$; $\nu \sim 5$).

The specificity of the parameters involved in building the model allows you to bring all values to a single monetary equivalent.

The differential equation of the model from (5) and its solution are given as follows:

$$\frac{dK(t)}{dt}=\sigma K(t)\qquad \text{де}\quad \sigma=\frac{\mu}{\nu}$$

$$K(t)=K_0 e^{\sigma t};\ \ I(t)=I_0 e^{\sigma t};\qquad Y(t)=Y_0 e^{\sigma t}$$

$$I_0=\sigma K_0;\quad Y_0=\frac{K_0}{\nu};\quad K_0=K(0);\quad I_0=I(o);\qquad Y_0=Y(0)$$

The financial flows of the system are discrete, which is why the above ratios are actually discrete. Then:

$$I_n=\mu\, Y_n;\qquad \frac{dK_n}{dt}=I_n\ ;\ K_n=\nu Y_n;\quad n=0,1,2,\ldots$$

In order to exclude the erroneous application of the operation of differentiation of functions of a discretely changing model, it is proposed to involve the apparatus of the theory of generalized functions [4, 7].

It is advisable to submit the law of capital formation in the form of:

$$K_n = K_0 + \sum_{j-1}^{n} I_j\ ,$$

$$\frac{dK(t)}{dt} = \sum_{j=0}^{n} I_j \delta(t-t_j)\,,$$

where $\delta(t)$ − the Dirac delta function.

Then the law of capital formation naturally turns from the arithmetic to the integral [11]:

$$K(t) = \int_{-\tau}^{t} I(s)\,ds = K_0 + K_R\ ,$$

where τ − the period of initial capital accumulation K_0, and $K_R = \int_0^t I(s)\,ds$.

Then the classical relation

$$dK(t)/dt = I(t),$$

makes ordinary sense.

Income, understood in the sense of flow intensity y(t), cannot be compared with capital in monetary equivalent K(t), which is provided discretely as annual incomes.

Let's compare the capital with the income realized during the period from 0 to t:

$$y_R(t) = \int_0^t y(s)\,ds.$$

A classic ratio $K_n = \nu y_n$ (n – year number) becomes integral:

$$K(t) = \nu \int_t^{t+1} y(s)\,ds, \quad t \geq 0 \tag{6}$$

Relation $K(t)/y_R(t)$ give ν if t=1; 2ν if $t = 1/2$; 3ν if $t = 1/3$ …..

For an arbitrary moment of time t, the ratio is equal to ν/t. Following the logic, instead of (6) we introduce the ratio

$$K(t) = \frac{\nu}{t} \int_0^t y(s)\,ds. \tag{7}$$

The peculiarity that arises when $t \to 0$, is eliminated by Lopital's rule and leads to the ratio

$$K_0 = \nu y_0.$$

So, we have the integral dependence of capital on income (7) and the ratio:

$$\frac{dK(t)}{dt} = I(t), \tag{8}$$

which represent Harrod's model in continuous form.

Substituting Y(t)=I(t)/μ in (7) and using (8), we find:

$$Y(t)=I(t)/\mu \quad K(t)=K_0/(1-\sigma t).$$

From where:

$$I(t)=\frac{I_0}{(1-\sigma t)^2}; \qquad Y(t)=Y_0/(1-\sigma t)^2.$$

Note that in the denominator of the fraction (7) t can be replaced by some function satisfying the conditions $f'(0) = 1$, $f(\nu(= \nu$.

$$K(t) = \frac{\nu}{f(t)} \int_0^t y(s)\,ds. \tag{7'}$$

Its presence in the mathematical model allows, in particular, to investigate the onset of the crisis, which is not foreseen by classical models.

This function can be extrapolated to the critical moment of time when $f(t_k) = \sigma^{-1}$.

In addition, in this case we have:

$$K(t)=K_0/(1-\sigma f(t))$$

and the onset of the crisis can be postponed by reducing the rate of capital accumulation.

In the Keynesian model, it is assumed that the GDP y(t+1) of the next year is equal to the aggregate demand of the previous (current) year. At the same time, the aggregate demand for consumer C and investment I goods depends only on the GDP of the current year:

$$y(t)=C[y(t)]+I(t) \tag{9}$$

Considering the linear dependence of demand for consumer goods, relation (9) at discrete time will take the form:

$$y(t+\Delta t)-y(t)=[C_{min}-(1-c)y(t)+I]\ \Delta t\ ,$$

where (1-c) − is the tendency to accumulate,

C_{min} − is the minimum volume of the accumulation fund.

When we come to a certain differential equation of the model.

As in the case of the Harrod-Domar model, the Keynes model, formally interpreted in categories of flow intensities, is discrete [1]. As well as for the Harrod model, for the Keynes model, the correction suggested by formula (7') should be made [5].

For economic relations, it is logical to assume the comparison of capital and total income in the form used in the mathematical description of a number of models

$$K(t)\sim \int_0^t Y(\eta)d\eta.$$

With reference to the first year of the period of economic growth, this leads to correlations

$$K_0=\nu Y_0; \qquad K(1)=\nu \int_0^1 Y(\eta)d\eta.$$

Or, as it was explained above, we get a dependency

$$K(t)=\frac{\nu}{t}\int_0^t Y(\eta)d\eta.$$

The condition at t=0 is fulfilled according to Lopital's rule, the units of measurement are dimensionless quantities. From the given relationship between income and capital, and the known equation

$$\frac{d\,K(t)}{dt}=I(t),$$

we get the following integral dependence between income and investments:

$$I(t)=-\frac{\nu}{t^2}\int_0^t y(s)ds+\frac{\nu}{t}y(t).$$

Omitting the mathematical calculations, we will give only the final form, for example, the equation of the Keynes model with a multiplier (an indicator of an additional addition to the input action) in the feedback loop:

$$T\frac{d\eta}{dt}+(1\mp\alpha)\eta=T\left[\frac{\nu}{t}\eta(t)-\frac{\nu}{t^2}\int_0^t \eta(s)ds\right], \tag{10}$$

η – GDP;

c – marginal propensity to consume;

1-c – extreme tendency to accumulate;

T/(1-c) – is the time constant of the inertia link;

α – is the multiplier gain indicator;

ν – is a dimensionless constant: the number of years for which annual income is balanced by capital.

It is important that both in the Keynes model and in the Harrod model, the Phillips model and other equations contain coefficients not as constants, but as functions of time (the right-hand side of equation (10)).

This significantly expands the possibilities of studying the behavior of economic systems and, in particular, in contrast to the classical model, for example, Harrod's model, allows to study the process of the emergence of crises that are interpreted. For example, as the occurrence of a collapse, a singular (special) solution to the given task, or the limitation of the time period for which forecasting can be carried out.

By differentiating equation (10) with respect to time and performing small transformations, we obtain a standard degenerate hypergeometric equation

$$\eta'' + \left(b + \frac{a}{t}\right)\eta' + \frac{c}{t}\eta = 0\,, \qquad (11)$$

where

$$(2\text{-}\nu)T=a; \quad 1\mp\propto=b; \quad 2(1\mp\propto)=c.$$

With certain numerical values of coefficients and ratios between them, the solution of equation (11) is a degenerate hypergeometric function, which, in particular, can turn into elementary ones, and further analysis of the behavior of the economic system is simplified. Setting, in this way, certain external input parameters of the system T, ν and α, it is possible to obtain the desired functioning of the system at the output.

So, for example, the analysis of the introduction of the multiplier into the model ($0 < \alpha < 1$) shows that the feedback (change in the coefficient b in the equation) leads to higher GDP growth, and therefore to growth in investment and consumption.

The solution of equation (11) can also be obtained using an operational method, for example, the Laplace transform in time [12]. It is important that the solution is presented in a closed analytical form.

In order to visually present the solution to equation (11), a numerical analysis was performed with unchanged initial data and variable parameters T, ν, and α. Figure 1 shows the change in GDP at α=0.5; T=2; ν=1 in Fig. 2 – at α=0.5; T=2; ν=1.8

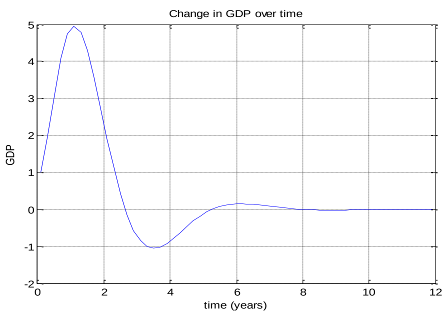

Figure 1. Dynamics of GDP
α=0.5; T=2; ν=1

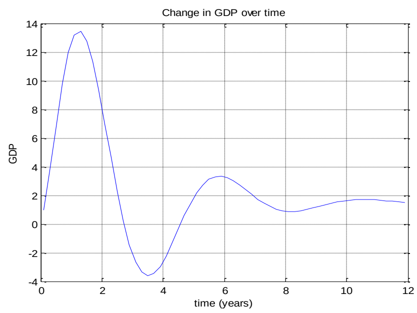

Figure 2 Dynamics of GDP
α=0.5; T=2; ν=1.8

Source: authors' own elaboration

There is a tendency of the guideline regime at a certain level of GDP. For this type of change in GDP, the parameter ν ≤ 2. As soon as, other parameters being equal, the parameter ν becomes more than 2.5, the change in GDP changes qualitatively.

Figure 3 shows the change in GDP when α=0.8; T=2; ν=3.

A characteristic sign of entering into a crisis is observed in Fig. 4, when α=0.8; T=2; ν=5

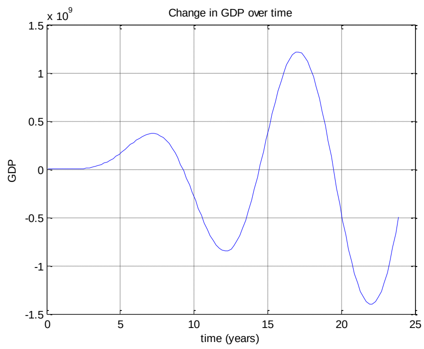

Figure 3. Dynamics of GDP
α=0.8; T=2; ν=3

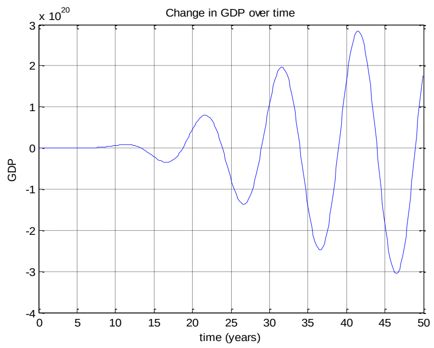

Figure 4. Dynamics of GDP
α=0.8; T=2; ν=5

Source: authors' own elaboration

Given certain initial conditions, equation (11) can be reduced to Voltaire's integral equation of the second kind [4], in which the presence of piecewise continuous functions does not cause significant difficulties in finding a solution. A number of advantages of describing processes with integral equations are known.

Note that a very important role in mathematical modeling of the economy is played by the dimension factor of the model components. An explanation of this is the classic Samuelson P.A. [2], when analyzing Harrod's model, look superficial. For him, issues of dimensionality when building models are "insignificant" and have a qualitative, ordinal character.

Let's give an example of inconsistency.

The direct use of the gradient operator of a function cannot be applied to the description of functions in a space of variables with different dimensions. For example, let the given function

$$F(hrv)=b_0(hrv)+B_1\left(\frac{hrv}{kil}\right)X_1(kil)+B_2(min)\ X_2\left(\frac{hrv}{min}\right),$$

where X_1, X_2 − natural variables,

B_0, B_1, B_2 − numerical parameters.

Then $grad(F)=\{B_1,B_2\}$ does not implement movement along the gradient, since the steps

$$X_{1n}(kil)=X_{01}(kil)+nB_1\left(\frac{hrv}{kil}\right)$$

$$X_{2n}\left(\frac{hrv}{kil}\right)=X_{02}\left(\frac{hrv}{kil}\right)+nB_2(min)$$

have dimensions that do not coincide with the dimensions of the variables.

You should go to the space of dimensionless (encoded) variables, for example:

$$F(hrv)=b_0(hrv)+b_1(hrv)x_1\left(\frac{б}{hrv}\right)+b_2(hrv)x_2\left(\frac{б}{hrv}\right)$$

And then the gradient steps take shape

$$X_{1n}(kil)=X_{01}(kil)+n\gamma\gamma\frac{1}{hrv})b_1(hrv)\Delta X_1(kil)$$

$$X_{2n}\left(\frac{hrv}{min}\right)=X_{02}\left(\frac{hrv}{min}\right)+n\gamma\gamma\frac{1}{hrv})b_2(hrv)\Delta X_2\left(\frac{hrv}{min}\right)$$

which allows you to realize the movement of the collinear gradient of the function F.

In our opinion, when building models, care should also be taken to observe the rules of action with the dimensions of the parameters included in the model description.

One of the most important directions in the analysis of economic systems is the construction and analysis of balance models, for example, Leontiev's intersectoral balance [6]:

$$x_t=A\ x_t+B(x_{t+1}-x_t)+c_t, \qquad (12)$$

where matrix A is the matrix of direct production costs;

B – matrix of incremental capital capacities;

c_t − is a column vector of final consumption.

The division of the gross output vector into three parts is shown:

Ax_t − current production consumption;

$B(x_{t+1}-x_t)$ − capital expenditures for production expansion;

c_t − final (non-production) consumption.

This discrete-time model is converted to a continuous-time model with an ordinary differential equation:

$$B(x_{t+1}-x_t)=(E-A)x_t-c_t,$$

$$B\big(x(t+\Delta t)-x(t)\big)=[(E-A)x(t)-c(t)]\Delta t, \qquad (13)$$

$$B\frac{dx}{dt}=(E-A)x-c(t)$$

In work [11], a transition from the system of linear algebraic equations resulting from the model of inter-branch balance (12) is proposed

$$x_i(t+t^*)=\sum_{j=1}^{n}a_{ij}x_j(t)+c_i; \qquad t\in[0,t^*],$$

to the system of differential equations by using expansion to the Taylor series with the transition to dimensionless time $\tau = t/t^*$:

$$d_\tau^2 x_i(\tau)+2d_\tau x_i(\tau)+2x_i(\tau)=2\sum_{j=1}^{n} a_{ij}x_j(\tau)+2c_i; \qquad \tau\epsilon[0,1]$$

The coefficients of the Taylor series, which are constants, are considered functions here. It is methodologically wrong in this case, when moving to an approximate model, to keep an arbitrary number of members of the series and to move to a differential equation of any order, replacing constants with differentials of functions. It is known, for example [6], that linear dynamic components of the system, performing different functions, are described by differential equations of different order.

It is clear that preserving one or another number of members of the Taylor series will not change the real functioning of the system, but its mathematical description is unlikely to correspond to reality.

In order to exclude the erroneous application of the differentiation operation to discretely variable functions of the model, it is better to involve the apparatus of the theory of generalized functions [12].

Conclusions

The possibility of unlimited (exponential) growth over an unlimited period of time follows from the mathematical formulation of a number of models of economic dynamics (for example, Harrod, Phillips, Solow models). It is assumed that this is caused, among other things, by the use of a continuous limit transition to deliberately discrete ratios.

In order to prevent contradictions in the mathematical modeling of economic dynamics, the integral dependence of capital on the intensity of income is reasonably proposed. At the same time, the models, as before, are described by differential equations, the coefficients of which, unlike the original model, are functions of time. A method of correcting contradictions arising from the incorrect use of macroeconomic functions of different dimensions in the built models is proposed.

Abstract

The analysis of main basic models describing macroeconomic processes leads to the conclusion that there are some inaccuracies in their formulation and, as a result, to erroneous conclusions about the behavior of the economic system. In the well-known Solow model, on a stationary development trajectory, the main economic indicators grow indefinitely (exponentially) in proportion to labor resources. In the Harrod-Domar model, the possibility of an economic crisis is structurally inevitable under the classical method of organizing the dependence of capital on the intensity of income. Similar inaccuracies arise in the models of Phillips and Keynes. It is proved that the incorrectness of the formulation and the possibility of unlimited macroeconomic growth at different time intervals appear in connection with the use of continuous analysis of infinitesimal to obviously discrete quantities. The methodology for constructing continuous mathematical models in natural science is based on the consideration of infinitesimal elements and the limiting transition, when the time interval tends to zero. A hypothesis is put forward that in economics such a method of transition to differential equations is devoid of objective meaning. It is precisely this application of the continuous analysis technique to knowingly discrete relations that leads to the contradictions that arise in the above-mentioned models. In order to construct a correct analogue of difference macroeconomic dynamic models, it is proposed to apply the integral dependence of capital on the intensity of income in the formulation of the problem. A correct approach to capital formation due to intensity flows is proposed. With this new dependence, the differential equations of the models, unlike the "classical" formulation, have time-dependent coefficients. This significantly expands the nature of the potential behavior of the economic system. With this new dependence, the differential equations of the models, unlike the "classical" formulation, have time-dependent coefficients. This significantly expands the nature of the potential behavior of the economic system. It becomes possible to determine the moment of the onset of the crisis. Based on this methodology, the transition from the classical formulation of Keynes's model to the model with the proposed integral relationship between income and investment is considered. A differential equation of the model has been obtained, which is a standard degenerate hypergeometric equation, the solutions of which, in particular, can be elementary functions. Graphical implementation and analysis of the solution for some values of the main parameters of the model are given. The inadequate exponential growth of the main indicators of the economy in the models under consideration has been eliminated.

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